EFFECT OF THERMAL STRESSESS ON THE MOTION OF A SIMPLE GAS

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On the basis of a gas-kinetic analysis it is shown that pure thermal stresses in a one-component gas lead to transfer proportional to the third power of the temperature gradient.

Let us consider a one-component gas in which a temperature gradient is somehow created in a time much less than the time of free travel of the gas molecules. It is assumed that the temperature gradient is constant over the entire gas volume. Let us determine how the presence of the temperature gradient will affect the gas motion. In doing this we will use the Boltzmann equation [1] for the distribution function f of the gas molecules:

$$\frac{\partial f}{\partial t}+(\vec{v}\nabla)f=I(f), \tag{1}$$

where I(f) is the collision integral and $\vec{v}$ is the velocity of the gas molecules.

We orient the y axis along the direction of temperature variation. Henceforth we will assume that the temperature gradient is sufficiently small, i.e.,

$$\varepsilon=\lambda\frac{\partial \ln T}{\partial y}\sim\frac{\lambda}{L}\ll 1, \tag{2}$$

where λ is the mean free path of the gas molecules, T is the absolute temperature, and L is a characteristic length parameter.

Owing to the fact that the temperature gradient is established rapidly enough we can assume that at the initial time the gas has a Maxwellian distribution,

$$f(t=0)\equiv f_0=n\left[\frac{m}{2\pi kT(y)}\right]^{3/2}\exp\left[-\frac{mv^2}{2kT(y)}\right], \tag{3}$$

where m is the mass of a molecule, n is the number of molecules per unit volume, and k is Boltzmann's constant.

Because of the smallness of ε [condition (2)] one can assert that the gas will relax to a Chapman–Enskog distribution [1]. The relaxation time will be on the order of the time of free travel. Therefore, one can seek the solution of the Boltzmann equation (1) with the initial condition (3) in the form

$$f=f_{\mathrm{C.E}}+\varphi(t)f_0 \tag{4}$$

where

$$f_{\mathrm{C.E}}=f_0\left[1-\frac{m}{kT}v_y\frac{t}{\rho}|\nabla p|+a_1\left(\frac{m}{2kT}\right)^{1/2}v_y\left(\frac{5}{2}-\frac{m}{2kT}v^2\right)\frac{|\nabla T|}{T}\right],$$

$$p=nkT.$$

Here the second term in the brackets describes the gas motion under the effect of the pressure gradient and is obtained by direct integration in a linear approximation of the Boltzmann equation. The third term corresponds to the Chapman–Enskog distribution in the field of the temperature gradient.

We introduce the designation

$$\vec{c}=\left(\frac{m}{2kT}\right)^{\frac{1}{2}}\vec{v}.$$

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With allowance for the initial condition (3), we obtain

$$\varphi(t=0) = -a_1 c_y\left(\frac{5}{2} - \vec{c}^2\right)\frac{|\nabla T|}{T}. \tag{5}$$

In a linear approximation with respect to ε we will seek the function $\varphi(t)$ in the form

$$\begin{cases} \varphi(t) = a(t) c_y\left(\dfrac{5}{2} - \vec{c}^2\right)\dfrac{\partial \ln T}{\partial y}, \\ a(0) = -a_1. \end{cases} \tag{6}$$

The latter equation follows immediately from the initial condition (5). In a linear approximation with respect to ε the Boltzmann equation is written in the form

$$\frac{\partial \varphi}{\partial t} = I(\varphi). \tag{7}$$

Actually, $\partial/\partial t \sim |\vec{v}|/\lambda$ and $\vec{v}\nabla \sim |\vec{v}|/L$. The ratio of the operators $(\partial/\partial t)/\vec{v}\nabla \sim L/\lambda \sim 1/\varepsilon$. Hence, it follows that $\partial/\partial t \gg \vec{v}\nabla$.

Multiplying both sides of Eq. (7) by $c_y(5/2 - \vec{c}^2)\exp(-\vec{c}^2)$ and integrating over velocity space, we obtain

$$\frac{\partial a}{\partial t} = -\frac{1}{\tau}a, \quad \tau = \left(\frac{m}{2kT}\right)^{\frac{1}{2}} a_1. \tag{8}$$

We integrate Eq. (8) with allowance for the initial condition (6):

$$a = -a_1 \exp\left(-\frac{t}{\tau}\right). \tag{9}$$

We substitute the solution obtained into the Boltzmann Eq. (1). We multiply both sides of the equation by $\vec{c}^2$ and integrate over the entire velocity space. As a result, we have

$$\frac{\partial T}{\partial t} = -u_{0y}\frac{\partial T}{\partial y} - \frac{2}{3}T \operatorname{div}\vec{u}_0 + \frac{\partial}{\partial y}\left\{\chi\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]\frac{\partial T}{\partial y}\right\}. \tag{10}$$

Here χ is the coefficient of thermal diffusivity and

$$\vec{u}_0 = -\frac{t}{\rho}\nabla p.$$

Let us consider a time $t \sim \tau$. Then on the right side of Eq. (10) one can replace $T(t)$ with $T(t=0) = T_0$, since the terms allowing for the variation in T over a time $t \sim \tau$ will be of a higher order of smallness.

Then the variation in T can be divided into two parts:

$$T(t) = T_{\text{hydr}}(t) + \Delta T(t), \tag{11}$$

where

$$T_{\text{hydr}} = T_0 - \int_0^t dt\left[u_y\frac{\partial T_0}{\partial y} + \frac{2}{3}T_0 \operatorname{div}\vec{u}_0 - \frac{\partial}{\partial y}\left(\chi\frac{\partial T_0}{\partial y}\right)\right],$$

while ΔT satisfies the equation

$$\frac{\partial \Delta T}{\partial t} = \left[-\frac{\partial \chi}{\partial T}\left(\frac{\partial T_0}{\partial y}\right)^2 - \frac{\chi}{\tau^2}\cdot\frac{\partial \tau}{\partial T}t\left(\frac{\partial T_0}{\partial y}\right)^2\right]\exp\left(-\frac{t}{\tau}\right). \tag{12}$$

The solution of Eq. (12) is written in the form

$$\Delta T(t) = A\left[\exp\left(-\frac{t}{\tau}\right) - 1\right] + Bt\exp\left(\frac{t}{\tau}\right), \tag{13}$$

$$\begin{cases} A = \tau\chi\dfrac{\partial \ln(\tau\chi)}{\partial T}\left(\dfrac{\partial T_0}{\partial y}\right)^2, \\ B = \chi\dfrac{\partial \ln \tau}{\partial T}\left(\dfrac{\partial T_0}{\partial y}\right)^2. \end{cases}$$

Thus,

$$T(t) = T_0\left\{1 - \frac{1}{T_0}\int_0^t dt\left[u_y\frac{\partial T_0}{\partial y} + \frac{2}{3}T_0 \operatorname{div}\vec{u}_0 - \frac{\partial}{\partial y}\left(\chi\frac{\partial T_0}{\partial y}\right)\right]\right.$$

$$\left. + \left[\frac{\tau\chi}{T_0}\cdot\frac{\partial\ln(\tau\chi)}{\partial T} + \frac{\chi t}{T_0}\cdot\frac{\partial\ln\tau}{\partial T}\right]\left(\frac{\partial T_0}{\partial y}\right)^2\exp\left(-\frac{t}{\tau}\right) - \frac{\tau\chi}{T_0}\cdot\frac{\partial\ln(\tau\chi)}{\partial T}\left(\frac{\partial T_0}{\partial y}\right)^2\right\}. \tag{14}$$

If we are not interested in the kinetic stage but only the hydrodynamic stage, then the initial condition of temperature variation for the hydrodynamic stage will be

$$T_{\text{hydr}}(t=0) = T(t=0) - \tau\chi\frac{\partial\ln(\tau\chi)}{\partial T}\left(\frac{\partial T_0}{\partial y}\right)^2. \tag{15}$$

We substitute the expression (14) obtained for T(t) into the Boltzmann Eq. (1) and allow for the smallness of ε (2). We then multiply both sides of the equation by v_y and integrate over velocity space. As a result, we obtain

$$\frac{\partial u_y}{\partial t} = -u_y\frac{\partial u_y}{\partial y} - \frac{1}{\rho}\nabla_y(nkT). \tag{16}$$

As in the case with the temperature, we can divide the variation in velocity with time into two stages: hydrodynamic and kinetic. The kinetic stage is characterized by the fact that it occurs for a time $\sim\tau$.

The kinetic stage is described by the equation

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}nk\frac{\partial}{\partial y}\left[A\exp\left(-\frac{t}{\tau}\right) + Bt\exp\left(-\frac{t}{\tau}\right)\right]. \tag{17}$$

The solution of Eq. (17) is written in the form

$$u = \frac{k}{m}(K + Mt + Nt^2)\exp\left(-\frac{t}{\tau}\right)\left(\frac{\partial T_0}{\partial y}\right)^3 + V, \tag{18}$$

$$K = 2\chi\left(\frac{\partial\tau}{\partial T}\right)^2 + 2\chi\tau\frac{\partial^2\tau}{\partial T^2} + \tau^2\frac{\partial^2\chi}{\partial T^2} + 4\tau\frac{\partial\tau}{\partial T}\cdot\frac{\partial\chi}{\partial T}, \tag{19}$$

$$M = \frac{2\chi}{\tau}\left(\frac{\partial\tau}{\partial T}\right)^2 + \chi\frac{\partial^2\tau}{\partial T^2} + 2\frac{\partial\chi}{\partial T}\cdot\frac{\partial\tau}{\partial T}, \tag{20}$$

$$N = \chi\left(\frac{\partial\ln\tau}{\partial T}\right)^2, \tag{21}$$

$$V = -\frac{k}{m}\left[2\chi\left(\frac{\partial\tau}{\partial T}\right)^2 + 2\chi\tau\frac{\partial^2\tau}{\partial T^2} + \tau^2\frac{\partial^2\chi}{\partial T^2} + 4\tau\frac{\partial\tau}{\partial T}\cdot\frac{\partial\chi}{\partial T}\right]\left(\frac{\partial T_0}{\partial y}\right)^3. \tag{22}$$

V is the velocity at the initial time in the hydrodynamic problem.

We assume that $\chi\sim T^{\alpha}$. Then [1] we have $\tau\sim T^{\alpha-1}$. Substituting χ and τ into (22), we obtain

$$V = F(\alpha)\frac{k}{m}\cdot\frac{\chi\tau^2}{T^2}\left(\frac{\partial T_0}{\partial y}\right)^3, \tag{23}$$

where

$$F(\alpha) = -9\alpha^2 + 10\alpha - 1.$$

In the model of a gas whose molecules interact with each other like rigid spheres $\alpha = 1/2$. Then

$$F(\alpha) = \frac{7}{4}.$$

If the molecules interact in accordance with the Lennard–Jones model, then at low temperatures $\alpha = 5/6$ and

$$F(\alpha) = \frac{13}{12}.$$

At high temperatures $\alpha = 2/3$ and

$$F(\alpha) = \frac{5}{3}.$$

Thus, the hydrodynamic velocity (23) arising at the initial time proves to be proportional to the third power of the temperature gradient, which permits one to ignore it in a linear approximation in problems of the kinetic theory of gases and in the physics of aerosols [2].

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UTILIZATION OF LIQUID–GAS BUBBLE MIXTURES FOR THE TRANSFER OF SHOCK-WAVE DISTURBANCES

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Transfer of the energy of shock-wave disturbances from a gas to a liquid or to a liquid–gas bubble mixture is considered. It is shown that the energy flux from the gas increases when a liquid with gas bubbles is substituted for the pure liquid.

Liquids have been used for a long time as the transmission medium for transmitting pressure from a gaseous medium to a certain object. For pressures at the wave front amounting to several hundreds of bars, most liquids behave as incompressible liquids. Therefore, as a shock wave from the gas reaches the liquid surface, the liquid receives only a weak acoustic wave carrying a small amount of stored energy, while most of the energy remains in the wave reflected from the liquid surface.

One of the possible ways to increase the percentage of the shock-wave energy transmitted to the liquid is to use a liquid mixed with gas bubbles as the transmission medium.

It is known [1] that the ratio of the wave energy flux in the liquid E_2 to the wave energy flux in the gas E_1 is determined by

$$\frac{E_2}{E_1} = \frac{4\rho_1 c_1 \cdot \rho_2 c_2}{(\rho_1 c_1 + \rho_2 c_2)^2}.$$

In this expression, ρ_1 and c_1 are the density and the sound velocity in the gas; and ρ_2 and c_2 are the density and the sound velocity in the liquid. If the acoustic resistances of two media are equal, complete transfer of energy from one medium to the other occurs, i.e., $E_2 = E_1$. If the acoustic resistance of one medium is much higher than the resistance of the other, we have $E_2 \ll E_1$ as the wave passes from the medium characterized by $\rho_1 c_1$ to the medium characterized by $\rho_2 c_2$. For instance, in the frequently encountered case of shock-wave transition at the air–water interface, $E_2 \approx 10^{-3} E_1$.

It is evident from the above relationship that the value of E_2 can be increased by reducing the density and the velocity of sound in the medium with the parameters $\rho_2 c_2$. For this, it is sufficient to use a liquid with gas bubbles instead of the pure liquid. For a liquid volume concentration up to 80%, the density of the two-phase medium ρ will change slightly, since

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